Classification
Physics Abstracts
05.40+j — 05.45+b — 05.70Jk

The Self-Organized Critical Forest-Fire Model with Trees and Bushes

B. Strocka (1), J.A.M.S. Duarte (2) and M. Schreckenberg (3)

(1) Institut für Theoretische Physik, Universität zu Köln, 50937 Köln, Germany
(2) Departamento Fisica, Faculdade de Ciencias, 4000 Porto, Portugal
(3) Theoretische Physik - FB 10, Universität Duisburg, 47048 Duisburg, Germany

(Received 2 May 1995, accepted 16 May 1995)

Abstract. — We present a forest-fire model with trees and bushes. Additional to the usual nearest-neighbour fire spreading both populations can ignite each other at the same lattice site. Simulations in two dimensions show that the bushes with the higher growth rate remain critical while the self-organization of the trees is destroyed by quick bush fires which ignite the tree clusters before they can become large. A mean-field treatment leads to coupled equations for the densities at the stationary point, depending on the ratio k of the growth rates.

In recent years the phenomenon of self organized criticality (SOC) has been attracting much interest. Many models are believed to show SOC, but the best understood so far is the sandpile model introduced by Bak, Tang and Wiesenfeld [1]. The forest-fire model is of special interest because it has no conservation laws. In its original form [2] it had only one parameter p, the probability that a tree grows at an empty site. This model was thought to show SOC in the limit $p \to 0$, but in simulations with rather small p spiral-shaped fire fronts occured with a typical length scale of order p^{-1} [3, 4]. It turned out to be necessary to introduce a lightning parameter f, a very small probability that a tree catches fire without having a burning neighbour. This model shows SOC under the condition of a separation of time scales for the burning down of clusters, the growing of new clusters and the time between two lightnings [5]. It is difficult to treat the model analytically and it has been solved exactly for one dimension only [6]. Scaling theory leads to relations between the critical exponents, which can be compared to the result of computer simulations [7–9]. It is also possible to introduce an immunity g so that the probability that a tree gets ignited by a burning neighbour becomes smaller than 1 [10,11]. Parallel, and even substantially predating these studies, the importance of two level systems – bushes and small vegetation near the ground and the tops of the trees on the upper level – has long been recognized in forest fire science. A mass of mainly empirical evidence has accumulated steadily and led to some sort of basic regimen classification [12,13]. But conclusions are hindered by the complexity of the phenomena themselves and the lack of adequate treatment of both the lower, surface fuels (subject to a wide variety of moisture and combustion parameters) and the crown canopy fuel separately. Despite these difficulties some consensus over the unlikelihood of surface-independent crown fires exists [12–14] although the principal mechanism for their sustained propagation varies in emphasis between the various researchers. Van Wagner's alternative, for example, hinges on a critical surface fire line intensity required for the fire to move to the crown level and adds propagation rates suitably calculated. Rothermel focusses on fire plume dynamics and air entrainment at the surface front, while Albini in a series of papers [15, 16] with strong wind conditions emphasizes the need for a fire spread model coupling both surface and crown fire effects. And mention should also be made of the independent theories developed by Grishin [17] through a detailed heat transfer balance system of differential equations. It is precisely such a consensus that brings a somewhat realistic slant to the model SOC as described above and constitutes the motivation for a basic, refinable two-level system. This can be improved upon by considering culling of the surface fuel under the tree canopy area, a procedure of great ecological importance [18], or by differentiating between the ignition conditions of both fuel levels. It is also appropriate to mention that the lightning probability parameter, so fruitfully introduced in Drossel and Schwabl, is indeed to be kept at a sufficiently small rate of occurrence, according to the system evolved by Fuquay *et al.* [19].

The forest-fire model is defined on a square or simple cubic lattice (more general on a d-dimensional hypercubic lattice) and each site can be empty, contain a tree or a burning tree. In the model we discuss another population, the bushes, live on the same lattice with the analogous options so each site can have 9 different states instead of three. The update is parallel for the whole lattice according to the following rules:

1. On a site without a tree, a tree grows with probability p. On a site without a bush, a bush grows with higher probability kp.

2. A burning tree or bush vanishes.

3. A tree becomes a burning tree if there is a burning bush at the same site or a burning tree at one of the neighbour sites. A bush catches fire if there is a burning tree at the same site or a burning bush at its neighbour sites.

4. A tree is ignited by lightning with probability f.

These rules without those for the bushes are exactly the Drossel-Schwabl model [5]. Our model has a new parameter $k \geq 1$ which determines the ratio of the growth speed of bushes and trees. If the bushes grow faster they can build larger clusters which, when burning down, disturb the self-organization of the tree clusters.

We have done computer simulations for a 1000×1000 square lattice with periodic boundary conditions. The ratio $\Theta = p/f$ of tree growth rate and lightning probability is always $\Theta = 10000$. The update was done step by step according to the rules and not with the algorithm that burns down entire clusters once they caught fire [9]. A cluster means here a connected number of trees or bushes, regardless of whether some of them are burning or not. After a long equilibration time, the sizes of bush and tree clusters were averaged over 2000 time steps.

In Figures 1 and 2, normalized cluster size distributions $N(s)$ are shown as a function of s. Figure 1 is the special case $k = 1$, in which trees and bushes can be considered as the same population. Although the cluster statistics were done for trees and bushes separately, the exponent remains the same as in the Drossel-Schwabl model $N(s) \propto s^{-\tau}$ with $\tau = 2.15(1)$. That is not too surprising because snapshots of the system for $k = 1$ (not shown) indicate that fire fronts and areas of dense forest fall together for both populations. In Figure 2 the

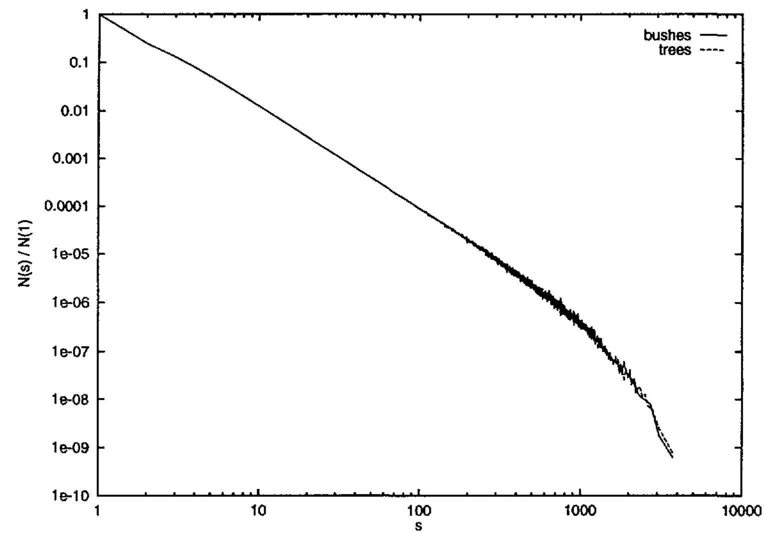


Fig. 1. — Normalized cluster size distribution of bushes and trees for $k = 1$.

tree cluster size distribution is plotted for different values of k compared to the one of the bushes, which essentially does not depend on k. The numbers of large tree clusters go down more rapidly for higher values of k. However, the power law for the trees still seems to hold for very small cluster sizes, but that cannot be proved from the simulations. Let $\rho_{\mathrm{t}}^t, \rho_{\mathrm{f}}^t$ and ρ_{e}^t denote the density of trees, burning trees and sites without trees at time t and $\sigma_{\mathrm{b}}^t, \sigma_{\mathrm{f}}^t, \sigma_{\mathrm{e}}^t$ for the bushes, respectively. The changes of the densities during one time step due to the update rules then read:

$$\begin{aligned}
\rho_{\mathrm{e}}^{t+1} - \rho_{\mathrm{e}}^t &= -p\rho_{\mathrm{e}}^t + \rho_{\mathrm{f}}^t && (1)\\
\rho_{\mathrm{f}}^{t+1} - \rho_{\mathrm{f}}^t &= -\rho_{\mathrm{f}}^t + (2d-1)\rho_{\mathrm{t}}^t\rho_{\mathrm{f}}^t + \rho_{\mathrm{t}}^t\sigma_{\mathrm{f}}^t && (2)\\
\rho_{\mathrm{t}}^{t+1} - \rho_{\mathrm{t}}^t &= -(2d-1)\rho_{\mathrm{t}}^t\rho_{\mathrm{f}}^t + p\rho_{\mathrm{e}}^t - \rho_{\mathrm{t}}^t\sigma_{\mathrm{f}}^t && (3)\\
\sigma_{\mathrm{e}}^{t+1} - \sigma_{\mathrm{e}}^t &= -kp\sigma_{\mathrm{e}}^t + \sigma_{\mathrm{f}}^t && (4)\\
\sigma_{\mathrm{f}}^{t+1} - \sigma_{\mathrm{f}}^t &= -\sigma_{\mathrm{f}}^t + (2d-1)\sigma_{\mathrm{b}}\sigma_{\mathrm{f}}^t + \sigma_{\mathrm{b}}\rho_{\mathrm{f}}^t && (5)\\
\sigma_{\mathrm{b}}^{t+1} - \sigma_{\mathrm{b}}^t &= -(2d-1)\sigma_{\mathrm{b}}^t\sigma_{\mathrm{f}}^t + p\sigma_{\mathrm{e}}^t - \sigma_{\mathrm{b}}^t\rho_{\mathrm{f}}^t. && (6)
\end{aligned}$$

For any initial state, after a transition period the system reaches a stationary state where the densities are constant. Then the left hand sides of equations (1)-(6) all equal zero. In this case using $\rho_{\mathrm{t}} + \rho_{\mathrm{f}} + \rho_{\mathrm{e}} = 1$ and $\sigma_{\mathrm{b}} + \sigma_{\mathrm{f}} + \sigma_{\mathrm{e}} = 1$, one obtains the following coupled equations for the densities $\rho_{\mathrm{t}}, \sigma_{\mathrm{b}}$:

$$\begin{aligned}
(2d-1)\rho_{\mathrm{t}}^2 - [2d + k(1-\sigma_{\mathrm{b}})]\rho_{\mathrm{t}} + 1 &= 0 && (7)\\
(2d-1)k\sigma_{\mathrm{b}}^2 - [2dk + 1 - \rho_{\mathrm{t}}]\sigma_{\mathrm{b}} + k &= 0. && (8)
\end{aligned}$$

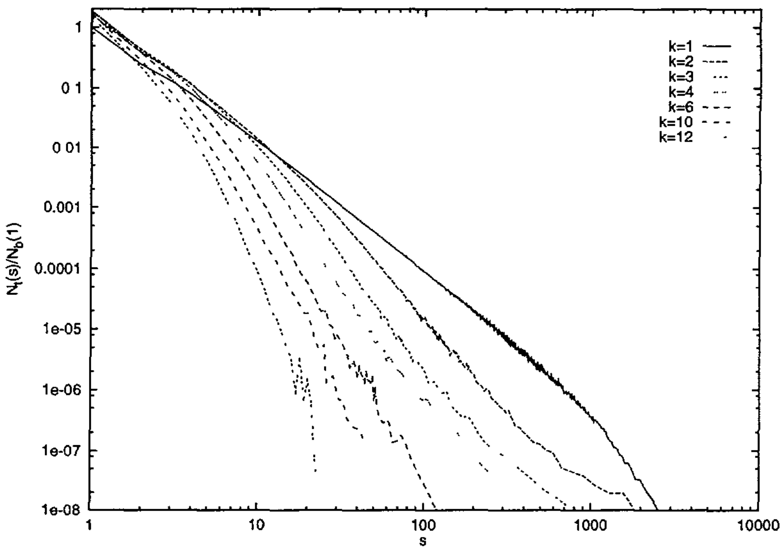
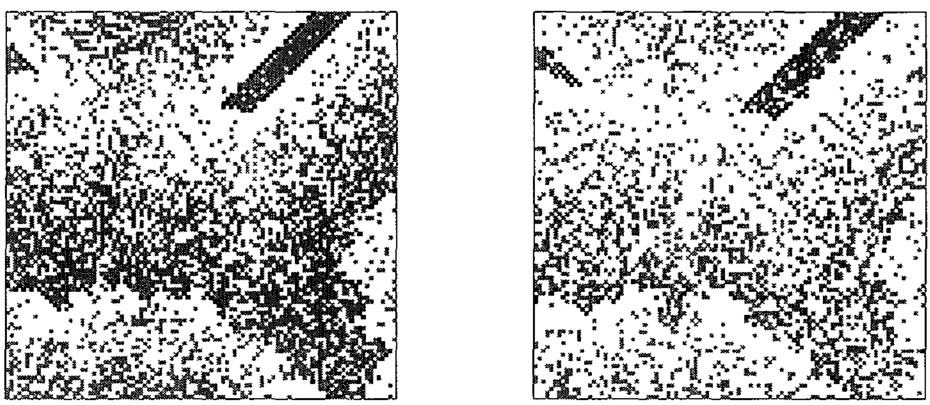


Fig. 2. — Cluster size distribution of the trees for different values of k, normalized by the number of single bushes $N_b(1)$.



a) b)

Fig. 3. — Snapshot of a 100×100 section of the system at $k = 2$, left hand: bushes, right hand: trees. Black dots indicate a burning, grey ones a non-burning and white ones an empty site.

The factor $(2d - 1)$ comes from the fact that a burning site at the time t was ignited by a burning neighbour at $t - 1$, so this neighbour site is empty at time t and the fire can only spread to the $(2d - 1)$ other sites. In Figure 3, a snapshot of a 100×100 area for both bushes and trees at the same point of time is given for $k = 2$. From this picture one can see that fire spreads typically in diagonal fire fronts so there are two burning sites igniting the same site, and both sites will be empty at the next timestep. Thus in two dimensions, the mean-field solution should become better if the factor $(2d-1)$ is replaced by $(2d-2)$ in all equations. By inserting equations (7),(8) into each other, one obtains equations of fourth order in ρ_t, σ_b. Due to the condition $f \ll p$, the lightning parameter f does not enter the mean-field equations (1)-(6). Hence the dense forest $\rho_t = \sigma_b = 1$ is a trivial solution of (7),(8) and can be eliminated in the equations for ρ_t and σ_b, leading to cubic equations. The relevant solution for the factor $(2d-1)$ read

$$\rho_t = \frac{1}{72}\left(41 + 5k + 2\sqrt{25k^2 + 266k + 241}\cos(\frac{\phi + 2\pi}{3})\right) \tag{9}$$

$$\text{with} \cos\phi = \frac{125k^3 + 1995k^2 + 5559k + 3689}{(25k^2 + 266k + 241)^{3/2}} \tag{10}$$

while for a factor $(2d-2)$ we get

$$\rho_t = \frac{1}{12}\left(k + 7 - \sqrt{k^2 + 14k + 1}\right), \tag{11}$$

respectively. The solution (10) is simpler because the trivial solution $\rho_t = 1$ is two-fold degenerate in the according equation. The update rules are symmetric in trees and bushes so the bush densities can be calculated from equations (9),(10) via

$$\sigma_b(k) = \rho_t(\frac{1}{k}). \tag{12}$$

In Figure 4 the solutions (9),(10) are shown together with simulation results for $d = 2$. The choice with the factor $(2d-2)$ fits the bush data better, while for high values of k the $(2d-1)$

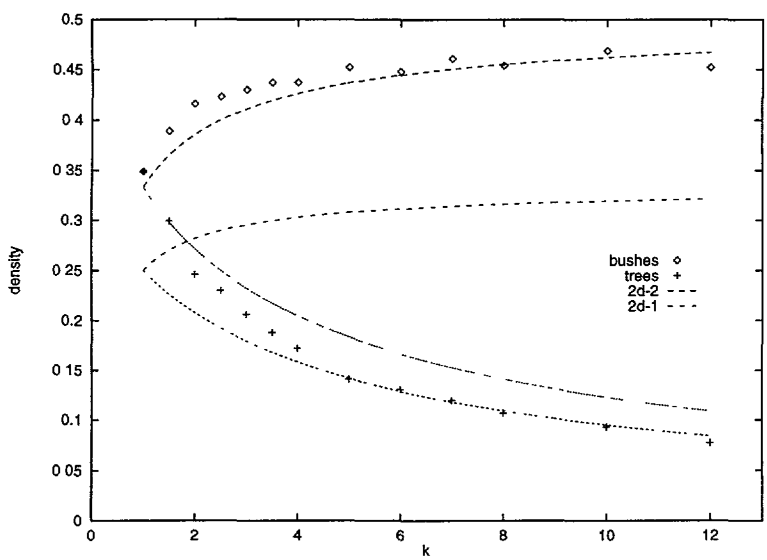


Fig. 4. — Density of bushes and trees for different values of k, together with the mean-field solutions for $(2d-2)$ (upper pair of lines) and $(2d-1)$.

solution is better for the trees. This is due to the fact that for high values of k the tree clusters become very small (Fig. 2), so the above assumption on diagonal fire fronts does no longer hold for the trees. Interestingly enough the mean-field solution fits the data quite well, although $d = 2$ is a very low dimension for using mean-field theory. In summary we have introduced a forest-fire model with two populations, which differ only in their growth rate. They are allowed to interact by igniting each other. Simulations show that the dominant bushes still retain the properties of the Drossel-Schwabl model, while the self-organization of the trees breaks down with increasing k. A mean-field theory considering the local fire-spreading leads to decent results even in two dimensions.

References

[1] Bak P., Tang C. and Wiesenfeld K., *Phys. Rev. Lett.* **59** (1987) 381.
[2] Bak P., Chen K. and Tang C., *Phys. Lett. A* **147** (1990) 297.
[3] Grassberger P. and Kantz H., *J. Stat. Phys.* **63** (1991) 685.
[4] Moßner W., Drossel B. and Schwabl F., *Physica A* **190** (1992) 205.
[5] Drossel B. and Schwabl F., *Phys. Rev. Lett.* **69** (1992) 1629.
[6] Drossel B., Clar S. and Schwabl F., *Phys. Rev. Lett.* **71** (1993) 3739.
[7] Grassberger P., *J. Phys. A* **26** (1993) 2081.
[8] Christensen K., Flyvberg H. and Olami Z., *Phys. Rev. Lett.* **71** (1993) 2737.
[9] Henley C.L., *Phys. Rev. Lett.* **71** (1993) 2741.
[10] Drossel B. and Schwabl F., *Physica A* **199** (1993) 183.
[11] Drossel B. and Schwabl F., *Physica A* **204** (1994) 212.
[12] van Wagner C.E., *Canad. J. Forest Res.* **7** (1977) 23.
[13] Williams F.A., *Prog. Energy Comb. Sci.* **8** (1982) 317.
[14] Rothermel R., Predicting the Behavior and Size of Crown Fires in the Northern Rocky Mountains, USDA-FS, Ogden UT (1990).
[15] Albini F.A., *J. Appl. Metereol.* **20** (1981) 1325.
[16] Albini F.A. and Stocks B.J., *Comb. Sci. Technol.* **48** (1986) 65.
[17] Grishin A.M., *Sov. Phys. Dokl.* **29** (1984) 917.
[18] Andre J., Gameiro Lopes A. and Viegas F.X., A Broad Synthesis of Research on Physical Aspects of Forest Fires 3, University of Coimbra (1992).
[19] Fuquay M., Baughman R.G. and Latham D.J., "A model for predicting lightning fire ignition in wildland fuels", USA-FS, Ogden UT, Res. Paper INT-217 (1979).